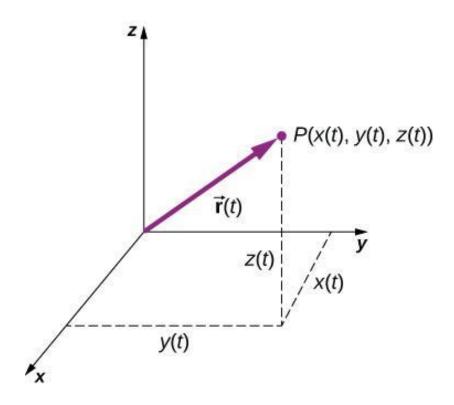




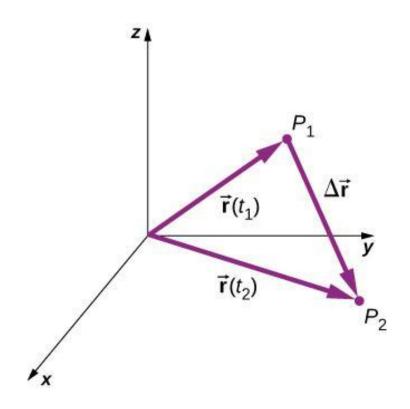
The Red Arrows is the aerobatics display team of Britain's Royal Air Force. Based in Lincolnshire, England, they perform precision flying shows at high speeds, which requires accurate measurement of position, velocity, and acceleration in three dimensions. (credit: modification of work by Phil Long)





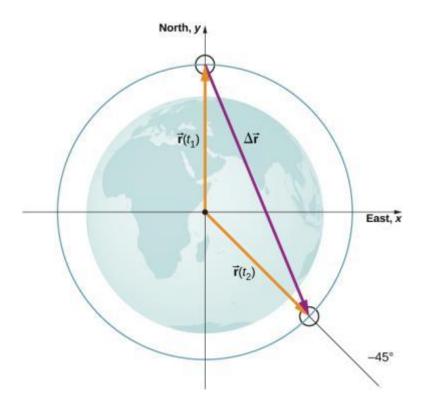
A three-dimensional coordinate system with a particle at position P(x(t), y(t), z(t)).





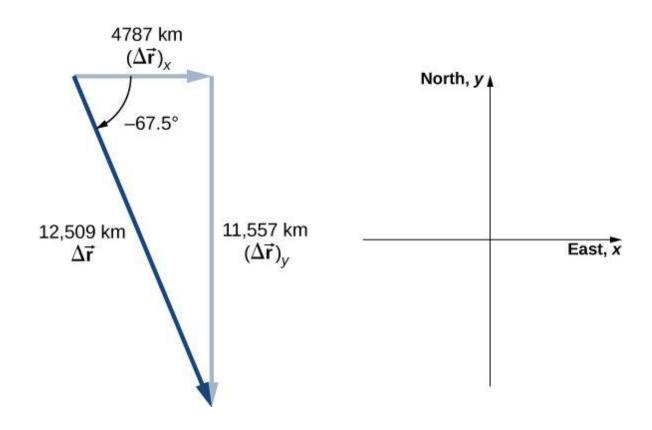
The displacement  $\Delta \vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$  is the vector from  $P_1$  to  $P_2$ .





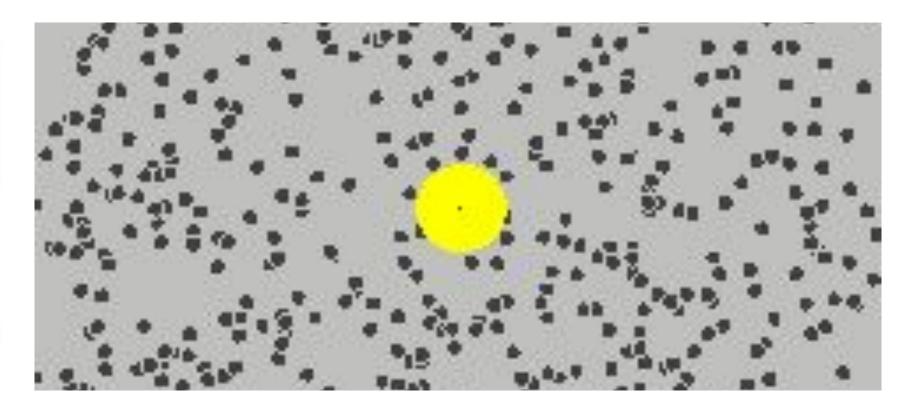
Two position vectors are drawn from the center of Earth, which is the origin of the coordinate system, with the *y*-axis as north and the *x*-axis as east. The vector between them is the displacement of the satellite.





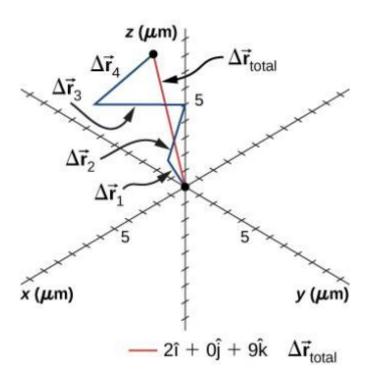
Displacement vector with components, angle, and magnitude.

# **Brownian Motion**



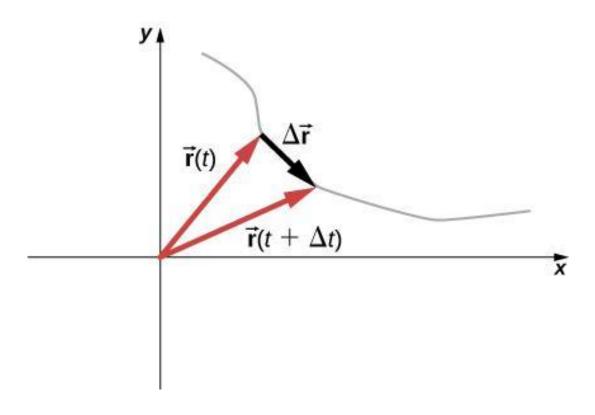
A heavy particle suspended in fluids moving randomly.





Trajectory of a particle undergoing random displacements of Brownian motion. The total displacement is shown in red.

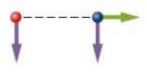


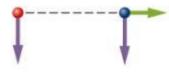


A particle moves along a path given by the gray line. In the limit as  $\Delta t$  approaches zero, the velocity vector becomes tangent to the path of the particle.

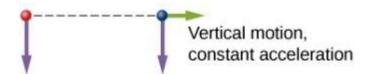






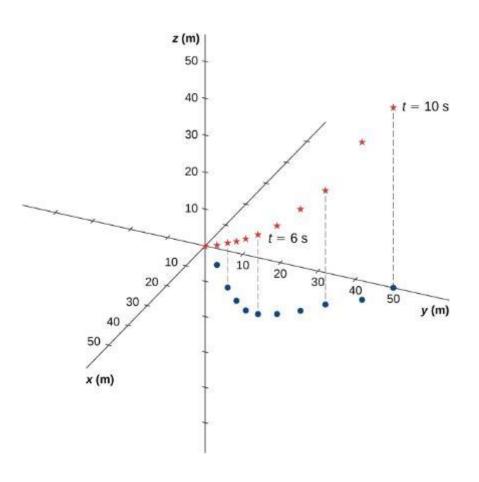


A diagram of the motions of two identical balls: one falls from rest and the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent the horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity whereas the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls, which shows the vertical and horizontal motions are independent.

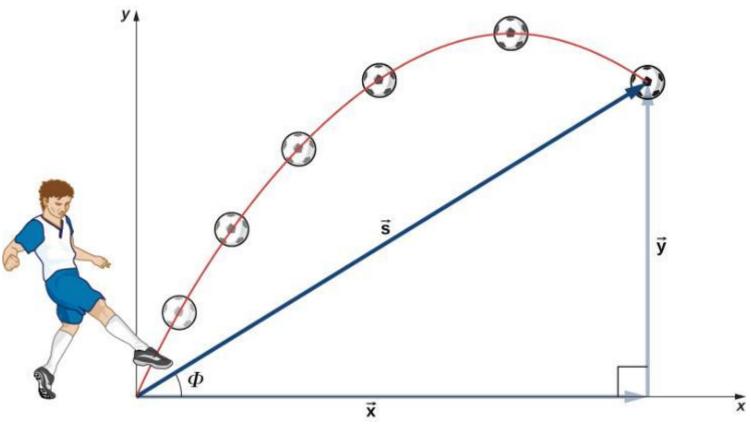




The particle starts at point (x, y, z) = (0, 0, 0) with position vector  $\vec{r} = 0$  as shown with red stars. The projection of the trajectory onto the xy-plane is shown blue dots. The values of y and z increase linearly as a function of time, whereas x has a turning point at t = 5 s and 25 m, when it reverses direction. At this point, the x component of the velocity becomes negative. At t = 10 s, the particle is back to 0 m in the x direction.

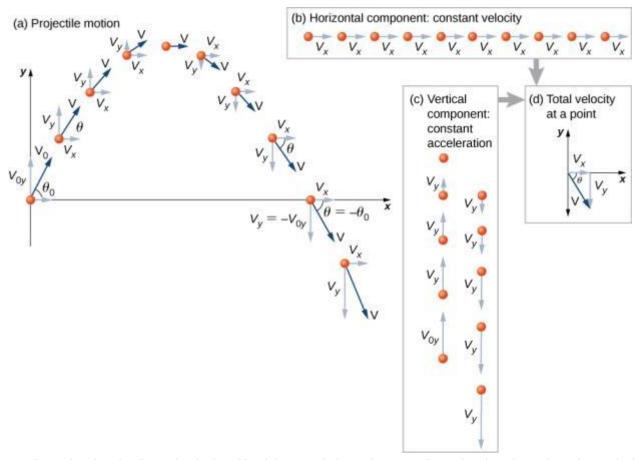






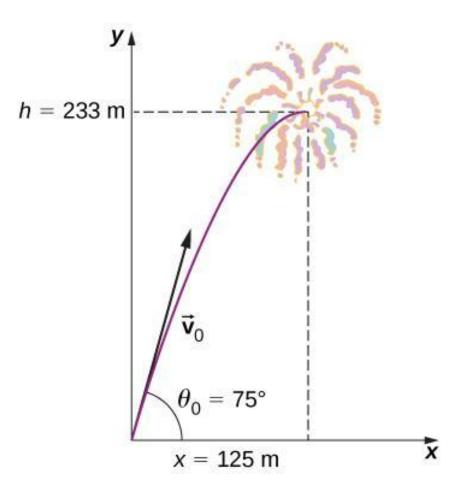
The total displacement s of a soccer ball at a point along its path. The vector  $\vec{s}$  has components  $\vec{x}$  and  $\vec{y}$  along the horizontal and vertical axes. Its magnitude is s and it makes an angle  $\theta$  with the horizontal.





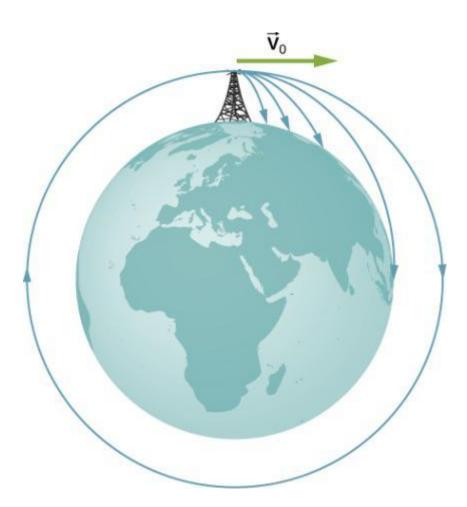
- (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes.
- (b) The horizontal motion is simple, because  $a_x = 0$  and  $v_x$  is a constant.
- (c) The velocity in the vertical direction begins to decrease as the object rises. At its highest point, the vertical velocity is zero. As the object falls toward Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity.
- (d) The x and y motions are recombined to give the total velocity at any given point on the trajectory.





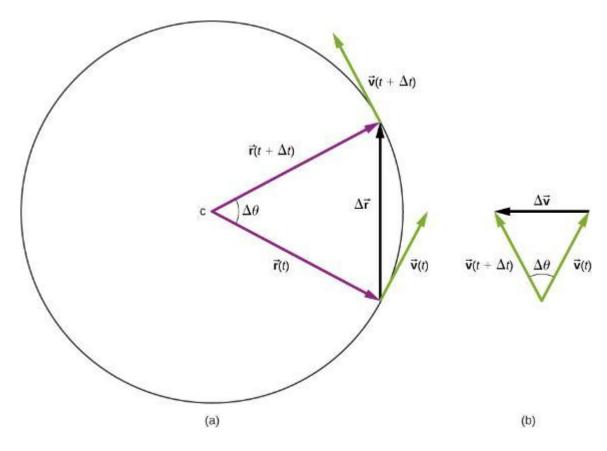
The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, which is found to be at a height of 233 m and 125 m away horizontally.





Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because Earth curves away beneath its path. With a speed of 8000 m/s, orbit is achieved.

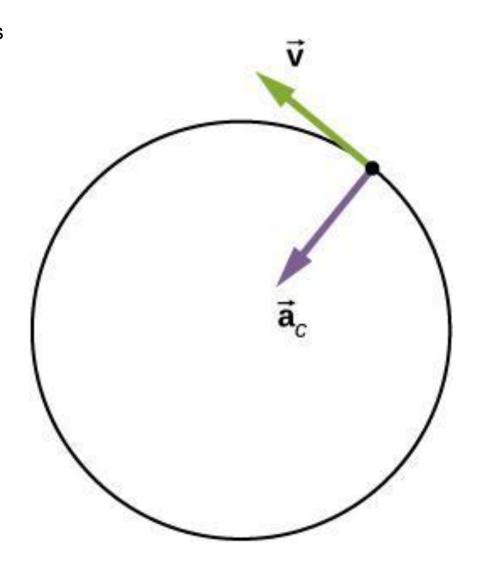




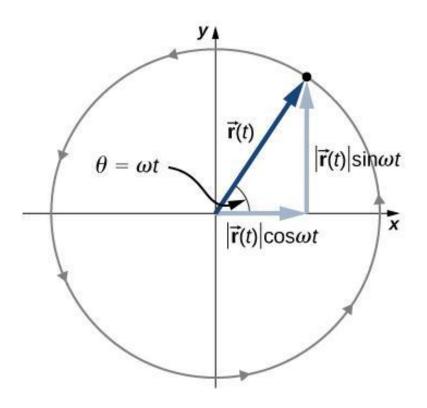
- (a) A particle is moving in a circle at a constant speed, with position and velocity vectors at times t and  $t + \Delta t$ .
- (b) Velocity vectors forming a triangle. The two triangles in the figure are similar. The vector  $\Delta \vec{v}$  points toward the center of the circle in the limit  $\Delta t \to 0$ .



The centripetal acceleration vector points toward the center of the circular path of motion and is an acceleration in the radial direction. The velocity vector is also shown and is tangent to the circle.



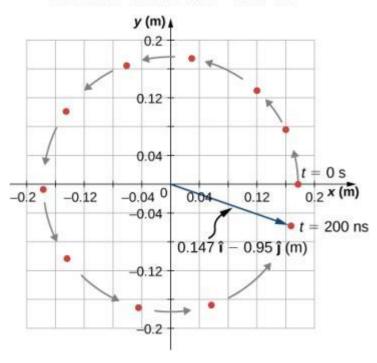




The position vector for a particle in circular motion with its components along the x- and y-axes. The particle moves counterclockwise. Angle  $\theta$  is the angular frequency  $\omega$  in radians per second multiplied by t.

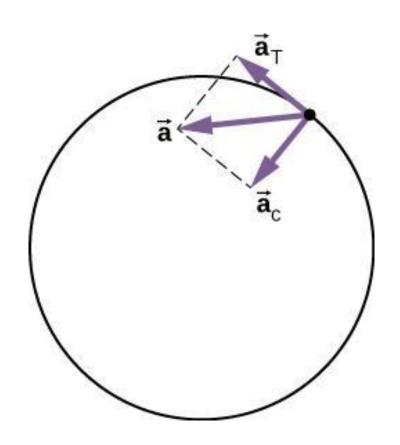


#### Position Vector at t = 200 ns



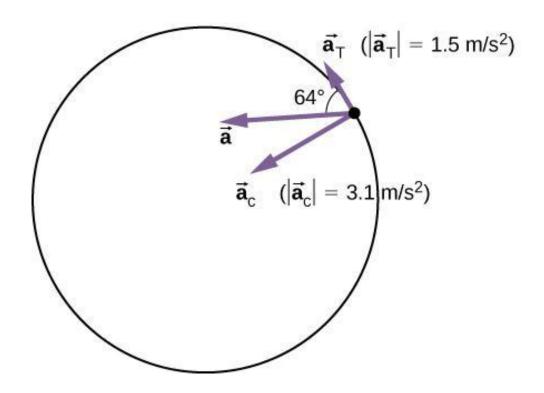
Position vector of the proton at  $t = 2.0 \times 10^{-7} \text{s} = 200 \text{ ns}$ . The trajectory of the proton is shown. The angle through which the proton travels along the circle is 5.712 rad, which a little less than one complete revolution.





The centripetal acceleration points toward the center of the circle. The tangential acceleration is tangential to the circle at the particle's position. The total acceleration is the vector sum of the tangential and centripetal accelerations, which are perpendicular.





The tangential and centripetal acceleration vectors. The net acceleration  $\vec{a}$  is the vector sum of the two accelerations.