Physics Notes for February 22

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Today, we saw in class the types of collision and how we can identify them. What I want to do here specifically is to show the cases of collisions and the conditions

- **Explosions (one to many)**
  A single object can explode into multiple objects (explosions).

$$\Delta \overrightarrow{\mathrm{P}} = 0 \ \& \ \Delta \mathrm{K} > 0$$

  The increase in the kinetic energy can be attributed to the potential energy the system has. For instance, in the cases of explosives, the chemical potential energy is converted into the kinetic energy of the shrapnel. The energy is not created out of thin air.

- **Inelastic Collisions**
  In this case, objects colliding have their momentum conserved but the kinetic energy decreases after impact.

$$\Delta \overrightarrow{\mathrm{P}} = 0 \ \& \ \Delta \mathrm{K} < 0$$

  A special case of inelastic collisions exists called *perfectly inelastic collisions* such that the kinetic energy after impact is the lowest possible in that situation and the colliding objects stick together(***many to one***).

- **Elastic Collisions(many to many)**
  In this case, different objects collide and the total kinetic energy of the system stays constant.

$$\Delta \overrightarrow{\mathrm{P}} = 0 \ \& \ \Delta \mathrm{K} = 0$$

  Perhaps, this is the one we need to discuss in a greater detail because the parameters are related in more than 1 manner. Since both the kinetic energy and linear momentum are conserved here, we have two starting equations to work with(assuming two colliding bodies):

$$\Delta \overrightarrow{\mathrm{P}} = 0 \implies \overrightarrow{\mathrm{P}}_i = \overrightarrow{\mathrm{P}}_f$$

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f} \tag{1}$$

$$\Delta \mathrm{K} = 0 \implies \mathrm{K}_i = \mathrm{K}_f$$

$$\frac{1}{2} m_1 v_{1,i}^2 + \frac{1}{2} m_2 v_{2,i}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2 \tag{2}$$

  Notice from equation (1) that we can rearrange the terms along with those on (2) to come up with the following equations

$$m_1 v_{1,i} - m_1 v_{1,f} = m_2 v_{2,f} - m_2 v_{2,i} \tag{3}$$

$$\frac{1}{2} m_1 v_{1,i}^2 - \frac{1}{2} m_1 v_{1,f}^2 = \frac{1}{2} m_2 v_{2,f}^2 - \frac{1}{2} m_2 v_{2,i}^2 \tag{4}$$

After taking common variables into consideration and multiplying both sides of (4) with 2, we get the following

$$m_1(v_{1,i} - v_{1,f}) = m_2(v_{2,f} - v_{2,i}) \tag{5}$$

$$m_1(v_{1,i}^2 - v_{1,f}^2) = m_2(v_{2,f}^2 - v_{2,i}^2) \tag{6}$$

Now that we have simplified the equations to where we can do simple algebraic computations, we can divide equation (6) by equation (5) to get the following result

$$\frac{v_{1,i}^2 - v_{1,f}^2}{v_{1,i} - v_{1,f}} = \frac{v_{2,f}^2 - v_{2,i}^2}{v_{2,f} - v_{2,i}} \tag{7}$$

Which is a very simple term on both sides

$$v_{1,i} + v_{1,f} = v_{2,f} + v_{2,i} \tag{8}$$

Now that we have equation (8), we can go back to equation (1) to substitute the values of the variables and find the final conditions after impact. Even better, the modified form of equation (8) can be modified to be

$$v_{2,f} = v_{1,i} + v_{1,f} - v_{2,i} \tag{9}$$

Now, let's put the equivalent of $v_{2,f}$ into equation (1), we get

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2(v_{1,i} + v_{1,f} - v_{2,i}) \tag{10}$$

Simplifying equation (10) as follows

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{1,i} + m_2 v_{1,f} - m_2 v_{2,i}$$

Taking the common variables into account, we get

$$(m_1 - m_2)v_{1,i} + 2m_2 v_{2,i} = (m_1 + m_2)v_{1,f}$$

Solving for $v_{1,f}$, we get the following

$$v_{1,f} = \frac{(m_1 - m_2)v_{1,i} + 2m_2 v_{2,i}}{m_1 + m_2} \tag{11}$$

Following the same steps above for $v_{2,f}$, we get the following equation

$$v_{1,f} = \frac{(m_2 - m_1)v_{2,i} + 2m_1 v_{1,i}}{m_1 + m_2} \tag{12}$$