St John Baptist De La Salle Catholic School, Addis Ababa

Grade 11 Physics Midterm Solutions

4th Quarter

May, 2024

This exam contains 16 questions, 7 pages (including the cover) for the total of 20 marks.

Conceptual Problems

1. (1 point) Two small objects each with a net charge of +Q exert a force of magnitude F on each other. We replace one of the objects with another whose net charge is +4Q. What is the magnitude of the force on the +Q charge now?

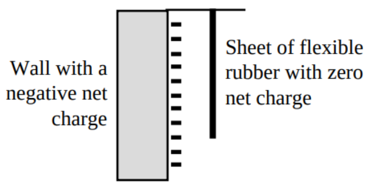
Answer

$$F_{new} = \frac{(Q)(4Q)}{4\pi\epsilon_0 r^2}$$

$$F_{new} = 4\frac{(Q)(Q)}{4\pi\epsilon_0 r^2}$$

$$F_{new} = 4F$$

2. (1 point) A non-conducting wall is given a negative net charge. Next, a sheet of very flexible rubber with zero net charge is suspended from the ceiling near the charged wall as shown below. What will happen to the rubber sheet?



Answer
The rubber sheet will bend toward the wall due to the polarization of the rubber molecules by the charged wall.

3. (2 points) You are given a problem involving a non-conducting sphere, centered at the origin. The sphere has a non-uniform, positive and finite volume charge density $\rho(r)$. You notice that another student has set the reference point for V such that $V = 0$ at the center of the sphere: $V(r = 0) = 0$. What would $V = 0$ at $r = 0$ imply about the sign of the potential at $r \to \infty$? Why?

Answer

We know that the potential difference between two points A and B is given by the following equation.

$$V_B - V_A = -\int_A^B \vec{E} \cdot \vec{dl}$$

And for simplicity and to define the absolute potential, we extended our definition to incorporate $r \to \infty$ and we modified it as below.

$$V(r) - V(\infty) = -\int_{\infty}^{r} \vec{E} \cdot \vec{dr}$$

We have defined the potential at infinity to be 0, thus the expression simply becomes

$$V(r) = -\int_{\infty}^{r} \vec{E} \cdot \vec{dr}$$

In this modified definition, however, we would need to redefine the boundaries of the integral and make them from 0 to r

$$V(\infty) - V(r) = -\int_{r}^{\infty} \vec{E} \cdot \vec{dr}$$

Since $V(r) = 0$ in the question, we would then simplify the equation above to the following rather simpler equation with one less term

$$V(\infty) = -\int_{r}^{\infty} \vec{E} \cdot \vec{dr}$$

For the insulating sphere, our potential becomes

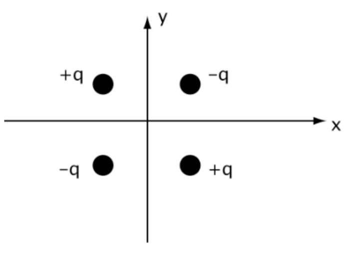
$$V(\infty) = \frac{-q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_r^{\infty}$$

$$V(\infty) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] \quad \left(\because \frac{1}{\infty} = 0\right)$$

$$V(\infty) = \frac{q}{4\pi\varepsilon_0 r}$$

As we can see above, it will be a positive value since the sphere was positively charged.

4. (2 points) You are given the following charge distribution made of 4 point charges, each located a distance "s" from the x- and y-axis. What is the dipole moment of the distribution?



Answer
The first thing we need to do is define the position vectors for each charge and then find the dipole moment about the origin for each charge and take the vector sum.

(I) For the -q charge in the **first quadrant** - since all the charges are in vertices of squares, we can safely assume a tilt of 45°. That means the position vector for the charge is simply $\vec{r_1} = \frac{s}{\sqrt{2}}(\hat{\imath}+\hat{\jmath})$. The dipole moment for this charge is in the $-\hat{r_1}$ direction due to the conventional definition of the direction of the dipole moment.

(II) By the same token, the position vector for the +q charge in the **second quadrant**, will be $\vec{r_2} = \frac{s}{\sqrt{2}}(-\hat{\imath}+\hat{\jmath})$. However, since it is a positive charge, the dipole moment will be in the same direction as the position vector here, which is $\vec{r_2}$.

(III) For the -q charge in the **third quadrant**, the position vector will be $\vec{r_3} = \frac{s}{\sqrt{2}}(-\hat{\imath}-\hat{\jmath})$. Since it is a negative charge, the direction of the dipole moment will actually be $-\vec{r_3}$.

(IV) For the +q charge in the **fourth quadrant**, the position vector will be $\vec{r_4} = \frac{s}{\sqrt{2}}(\hat{\imath}-\hat{\jmath})$. Since it is a positive charge, the direction of the dipole moment will be $\vec{r_4}$.

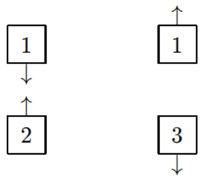
The dipole moment of the distribution will then be

$$\vec{p} = q(-\vec{r_1}+\vec{r_2}-\vec{r_3}+\vec{r_4})$$

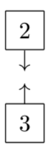
$$\vec{p} = q(-\frac{s}{\sqrt{2}}(\hat{\imath}+\hat{\jmath}) + \frac{s}{\sqrt{2}}(-\hat{\imath}+\hat{\jmath}) - \frac{s}{\sqrt{2}}(-\hat{\imath}-\hat{\jmath}) + \frac{s}{\sqrt{2}}(\hat{\imath}-\hat{\jmath})$$

We can see that the dipole distribution of the system simplifies to 0.

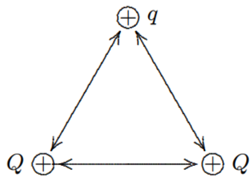
5. (1 point) The diagram shows two pairs of heavily charged plastic cubes. Cubes 1 and 2 attract each other and cubes 1 and 3 repel each other. Illustrate the forces of cube 2 on cube 3 and cube 3 on cube 2?



Answer
We can infer that 1 and 2 are unlike charges while 1 and 3 are like charges, so 2 and 3 will attract since charge 3 is the same kind as charge 1. So it will be as shown below



6. (2 points) Two particles, each with charge Q, and a third particle, with charge q, are placed at the vertices of an equilateral triangle of side length l as shown below. What is the net electric field at the point where q is found? What is the magnitude and direction of the total force on the particle with charge q? (in terms of Q and q)



Answer

The first part that simplifies our answer is that since this is simply an equilateral triangle and the charges on the bottom are the same, we can use symmetry to simplify the forces present. We can logically infer that any charge placed at the top vertex of this triangle will only experience a force vertically upwards. That is because the bottom left charge exerts a force that has components to the **right** and **vertically up** while the bottom right charge exerts a force to the **left** and **vertically up**. Since the charges have the same magnitude and the arrangement is an equilateral triangle, the right and left parts cancel out since force is a vector.

The second thing we need to do is find the Y component of each force. That is simply the force multiplied by the sine of 60°. To find the net force, we multiply by two. ***Why?***

$$F_{net} = 2\frac{Qq}{4\pi\epsilon_0 l^2}(\sin 60°)\hat{j}$$

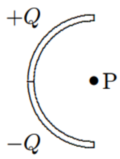
$$\vec{F}_{net} = \frac{\sqrt{3}Qq}{4\pi\epsilon_0 l^2}\hat{j}$$

Since we have found the force, we can find the electric field.

$$\vec{E} = \frac{\vec{F}}{q} \implies \vec{E} = \frac{\frac{\sqrt{3}Qq}{4\pi\epsilon_0 l^2}}{q}\hat{j}$$

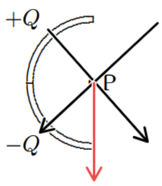
$$\vec{E} = \frac{\sqrt{3}Q}{4\pi\epsilon_0 l^2}\hat{j}$$

7. (1 point) Positive charge +Q is uniformly distributed on the upper half a semicircular rod and negative charge -Q is uniformly distributed on the lower half. What is the direction of the electric field at point P, the center of the semicircle?



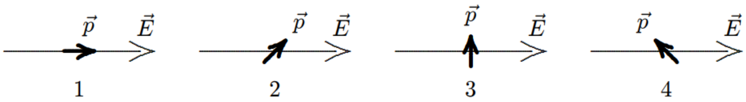
Answer

We can simply take out a line of electric field from each quarter of the semicircle and draw the lines. We can see how it turns out below



Similar to the question above, we have equal charges so the horizontal component of the field cancels out and we have a net field **vertically downwards** - the resultant is shown above by the red arrow.

8. (1 point) The diagrams show four possible orientations of an electric dipole in a uniform electric field $\vec{E}$. Rank them according to the magnitude of the torque exerted on the dipole by the field, least to greatest.



Answer
We know that $\tau = \vec{p} \times \vec{E} = pE\sin\theta$. So, the nearer the angle between them is being to 90 degrees, the larger the torque. The arrangement is given below

1, 2 and 4 tie, then 3

Problems

9. (1 point) The dipole moment of a dipole in a 300 N/C electric field is initially perpendicular to the field, but it rotates so it is in the same direction as the field. If the moment has a magnitude of $2 \times 10^{-9} C \cdot m$, what is the work done by the field?

Answer

$$W = \vec{p} \cdot \vec{E}, \text{ but since the the moment and electric field are parallel,}$$
$$W = pE = (2 \times 10^{-9} C \cdot m)(300N/C) = 6 \times 10^{-7} J$$

10. (2 points) A 5.0-C charge is 10 m from a -2.0-C charge. What is the electric field by the negative charge? What is the direction and magnitude of the electrostatic force on the positive charge?

Answer

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \text{ such that } \hat{r} \text{ is vector radially outward from the -2C charge and joins the 5C charge}$$

$$\vec{E} = \frac{-2C}{4\pi\epsilon_0 (10m)^2}\hat{r} = -1.80 \times 10^8 \hat{r} \text{ N/C}$$

Finding the force is simple after we have found the electric field.

$$\vec{F} = q\vec{E} = (5C)(-1.80 \times 10^8 \hat{r} N/C) = -8.99 \times 10^8 \hat{r} \text{ N}$$

11. (1 point) What is the flux of the electric field $(10N/C)\hat{\imath} + (20N/C)\hat{\jmath} + (8N/C)\hat{k}$ through a $2.0m^2$ portion of the yz plane?

Answer

$$\Phi_E = \vec{E} \cdot \vec{A} = ((10N/C)\hat{\imath} + (20N/C)\hat{\jmath} + (8N/C)\hat{k}) \cdot (2.0m^2)\hat{\imath}$$
$$\Phi_E = 20Nm^2/C$$

12. (1 point) A particle with a charge of 5.5μ C is 3.5 cm from a particle with a charge of -40.0μC. What is the potential energy of this two-particle system, relative to the potential energy at infinite separation?

Answer

$$U = \frac{Qq}{4\pi\epsilon_0 r} = \frac{(5.5\mu C)(-40.0\mu C)}{4\pi\epsilon_0 (3.5cm)} = -56.5J$$

13. (1 point) Explain why one of the following is the correct statement. *Simply choosing the answer has no value. The explanation is mandatory.*

A. A proton tends to go from a region of low potential to a region of high potential

B. The potential of a negatively charged conductor must be negative

C. If $\vec{E} = 0$ at a point P then V must be zero at P

D. If $V = 0$ at a point P then $\vec{E}$ must be zero at P

E. None of the above are correct

Answer
E

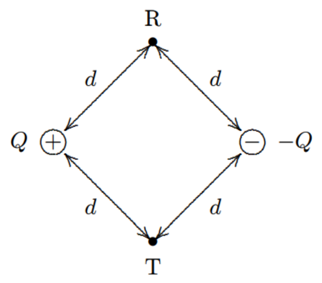
14. (1 point) The potential difference between two points is 100 V. If a particle with a charge of 4 C is transported from one of these points to the other, what is the magnitude of the work done? If the distance between the two points is 2 m, what is the magnitude of the electric field established?

Answer

$$|W| = |-q\Delta V| = 4C(100V) = 400J$$

$$|E| = \frac{\Delta V}{d} = \frac{100V}{2m} = 50V/m$$

15. (1 point) Points R and T are each a distance d from each of two particles with charges of equal magnitudes and opposite signs as shown. What is the work required to move a particle with a negative charge q from R to T?



Answer
Let's calculate the potentials at the points R and T.

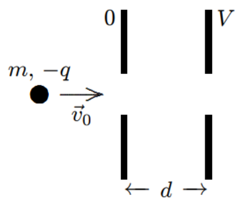
$$V_R = \frac{Q}{4\pi\epsilon_0 d} + \frac{-Q}{4\pi\epsilon_0 d} \;\&\; V_T = \frac{Q}{4\pi\epsilon_0 d} + \frac{-Q}{4\pi\epsilon_0 d}$$

We can see that both V_R and V_T are 0.

$$W = -q\Delta V = -(-q)(V_T - V_R) = 0$$

So, no net work done is needed to move a charge from point R to T.

16. (1 point) A particle with mass m and charge $-q$ is projected with speed v_0 into the region between two parallel plates as shown. The potential difference between the two plates is V and their separation is d. What is the change in kinetic energy of the particle as it traverses this region?



Answer
The only force the particle experiences is the electric force which means that the mechanical energy is conserved. Thus, we have the following

$$\Delta ME = 0 \implies \Delta K + \Delta U = 0 \implies \Delta K = -\Delta U$$

But, we also know that $-\Delta U = W$ for conservative forces. So, simply the change in the kinetic energy of the particle is the work done by the electric field.

$$\Delta K = W = -q\Delta V = -(-q)(V - 0) = qV$$

We can see that it accelerates(as we would expect) because the change in kinetic energy is positive.

Optional Problem

* Positive charge Q is distributed uniformly throughout an insulating sphere of radius R, centered at the origin. A particle with positive charge Q is placed at x = 2R on the x axis. Show that the magnitude of the electric field at x = R/2 on the x axis is $\dfrac{Q}{72\pi\epsilon_0 R^2}$

Answer
The net electric field has two components at R/2, an outward directed field from the uniform charge and an inward pointing field from the charge. The field from the charged ball at the surface is $\dfrac{Q}{4\pi\epsilon_0 R^2}$. Since only 1/8th (**Why?**) of the full charge is inside at r= R/2, the field then becomes $E_1 = \dfrac{\frac{Q}{8}}{4\pi\epsilon_0(\frac{R}{2})^2} = \dfrac{Q}{8\pi\epsilon_0 R^2}$.

The contribution from the single charge at R/2 is $E = \dfrac{Q}{4\pi\epsilon_0(2R - \dfrac{R}{2})^2} = \dfrac{Q}{9\pi\epsilon_0 R^2}$.

The net electric field then becomes

$$\vec{E}_{net} = \frac{Q}{8\pi\epsilon_0 R^2}\hat{i} - \frac{Q}{9\pi\epsilon_0 R^2}\hat{i} = \frac{Q}{72\pi\epsilon_0 R^2}\hat{i}$$